

INTERDISCIPLINARY GREEK CURRICULUM FRAMEWORK FOR MATHEMATICS

1. Teaching aim

The aim of teaching mathematics, which can be placed among the general aims of school education, is to facilitate the students' personal development and provide them with the necessary skills for their smooth social integration. Mathematics can help students develop structured and critical thinking abilities and improve their reasoning abilities of analysis, abstraction and generalization that will enable them to express their thoughts in a neat, clear, simple, and accurate way.

Mathematics also sharpens students' abilities of observation, self-concentration and persistence, stimulates their initiative, creative imagination and freethinking, and fosters their sense of harmony, order, and beauty.

Mathematics is a necessary tool in everyday life, especially at the workplace. Mathematics also has a significant contribution to the development of other scientific fields, especially in Technology, Economics, and Social Studies.

2. Content Guiding Principles, Goals, and Fundamental Concepts of Interdisciplinary Approaches

I. ELEMENTARY SCHOOL

Grade	Content Guiding Principles	General Goals (knowledge, skills, attitudes, and values)	Indicative Fundamental Interdisciplinary Approaches
1 st , 2 nd , 3 rd , 4 th , 5 th , 6 th	Problem Solving	Students must be able to: explore mathematical situations; pose problems and formulate questions using a set of given data; rephrase problems; recognize and describe similar problems and mathematical situations; investigate open-ended questions; apply their mathematical skills in everyday situations; and learn how to use calculators, computers, internet, and other recourses	Change Interaction System Communication Individual-group (element-set) Similarity-Difference
1 st	Numbers and Operations	Students must be able to: count orally, read, write, and order natural numbers up to 100; add numbers not greater than 20; subtract numbers not greater than 20; become familiar with situations that entail multiplication and division, such as equal grouping of object and sharing equally	Change Communication Individual-group (element-set) Similarity-Difference
1 st	Measurements	become familiar with the concepts of length, time, money, and mass; recognize, describe, extend, and generalize numeric and geometric patterns	System Space-Time Similarity-Difference
1 st	Geometry	practice and develop spatial orientation; draw, reproduce, recognize, name, and classify geometrical figures; identify solids: cubes, right rectangular prisms, cylinders, spheres; recognize reflective symmetry in pictures and figures.	Change System Communication Space-Time Similarity-Difference

GREEK CURRICULUM IN MATHEMATICS

ELEMENTARY SCHOOL

1. Particular goals

Students might be able to:

Construct elementary mathematical knowledge and abilities;

Communicate using mathematical language;

Understand elementary mathematical methods;

Develop critical thinking processes and proving abilities;

Develop problem solving ability;

Apply and use mathematics in everyday problems;

Understand the evolution of mathematics (the historical evolution of mathematical tools, symbols, and concepts)

Develop positive attitude towards mathematics

2. Goals, Topics, and Indicative activities

1st Grade

Goals	Topics (estimation of time)	Indicative activities
<p>Students must be able to:</p> <p>Activate their already existing knowledge;</p> <p>Check and verify results of operations.</p>	<p><i>Problem solving</i></p> <p><i>(18 hours)</i></p>	<p>Introduction of activities where students are participant learners;</p> <p><i>Solve problems through dramatization, games or about real life transactions.</i></p>
<p>Compare sets of discrete objects counting them or use one-to-one correspondence between objects;</p> <p>Write the numbers from 0 up to 10;</p> <p>Count orally up to 10;</p> <p>Skip-count orally by twos up to 10;</p> <p>Count sets of objects that contain at most 10 objects;</p> <p>Read the arithmetic symbols;</p> <p>Count down from 10 to 0;</p> <p>Figure out the antecedent and the following number of any number up to 10;</p> <p>Recognize quantities of one, two, or three elements fast (direct estimation);</p> <p>Break down in sums all the numbers up to 5;</p> <p>Calculate sums up to 5;</p> <p>Solve problems of addition using the symbols of “+” and “=”;</p>	<p><i>Numbers and operations</i></p> <p>Calculate up to 5;</p> <p>Count up to 10;</p> <p>The symbol of equation “=”;</p> <p>The symbol of addition “+”</p> <p><i>(25 hours)</i></p>	<p>Count the elements of a set of objects;</p> <p>Read one-digit numbers on cards or on the number line;</p> <p>Fill in the missed numbers in an arithmetic sequence or continue a given sequence of numbers;</p> <p>Split up a set of objects;</p> <p>Exchange euros up to 5€;</p> <p>Unify sets of elements;</p> <p>Represent numbers using objects or dice or digits;</p> <p>Introduce problems and activities to symbolize addition and equation.</p> <p><i>Create collections of objects and count them. Play games where students sell and buy things (i.e., monopoly)</i></p>

<p>Represent using objects and break down the numbers from 6 to 10 using fives or doubles (n+n);</p> <p>Recognize, read, and write the numbers up to 20;</p> <p>Verify the results of addition through subtraction and the results of the subtraction through addition;</p> <p>Skip-count orally by fives and by tens up to 20;</p> <p>Compare numbers using the appropriate symbol of inequality;</p> <p>Break down in sums the numbers up to 10 using fives and doubles (n+n);</p> <p>Use alternative presentations of calculations emerging the commutative property of addition;</p> <p>Subtract using the symbol of subtraction “-“</p>	<p><i>Numbers and operations</i></p> <p>Calculate up to 10;</p> <p>Count up to 20;</p> <p>Use the symbols “=”, “<”, “>”</p> <p>(25 hours)</p>	<p>Represent for example 7 as 5+2, or 8 as 5+3 and 4+4;</p> <p>Use the number line;</p> <p>Use empirical strategies like figures or verbal expressions;</p> <p>Introduce activities that present for example the number 6 as 5+1, or 8 as 5+3 etc.;</p> <p>Express 10 using a variety of sums;</p> <p>Introduce subtraction using in the activities the symbol $a - b = *$ and not the symbols $a - * = b$ or $* - a = b$;</p> <p><i>Students could draw the members of their family adding more relatives (grandfathers, grandmothers, aunts, uncles, cousins, etc.), family friends, classmates etc. Students could count the females, the males, and the children. They could subtract from the number of adults the number of children.</i></p>
<p>Add numbers up to 20;</p> <p>Subtract numbers up to 20;</p> <p>Check and verify the results of addition and subtraction through the inverse operation;</p> <p>Skip-count orally by tens up to 50 and then up to 100;</p> <p>Use the properties of addition to express the sums as follows: $8+5=(8+2)+3=10+3=13$ $8+7=8+(8-1)=(8+8)-1=16-1=15$;</p> <p>Represent the two-digit numbers in the decimal system;</p> <p>Use the decimal form of numbers to calculate a) their sum and b) their difference;</p> <p>Represent two-digit numbers and calculate their difference;</p> <p>To get acquainted with situations of repetition of equal quantities and sharing equally without introducing the symbols of multiplication and division.</p>	<p><i>Numbers and operations</i></p> <p>Make calculations up to 20;</p> <p>Count up to 50 and then up to 100;</p> <p>Use situations of repletion of equal quantities and sharing equally.</p> <p>(22 hours)</p>	<p>Students use dice or abacus to represent quantities;</p> <p>Students calculate for example 6+5 as 5+1+5 or 9+7=9+1+6 etc.;</p> <p>Students use abacus or other teaching means to express the tens and the ones of numbers.</p> <p>For example: $23+5=10+10+3+5$ $18-3=10+8-3$;</p> <p>Students play with cards which contain pictures, words, and symbols to calculate up to 20, to count up to 100, and to use the symbols “=”, “<”, and “>”;</p> <p>Each student skip-counts on the number line by twos to understand the concept of “times”;</p> <p><i>Students visit and draw a forest with trees, bushes, flowers etc. They count up to 100, they use tables with numbers, they calculate up to 20 etc.</i></p>

<p>Measure different magnitudes using known units of measurement or using arbitrary units of measurement; Compare the magnitude or the dimensions of two or more objects using the expressions: taller than ... shorter than ... narrower than ... wider than ... etc; Present chronological order of events; Estimate and distinguish the duration of timeframes; Distinguish by experience the different types of coins according to students' knowledge about numbers; Figure out relationships between coins; Estimate the value of the coins; Understand the function of balance; Use the expressions: heavier than ... lighter than ...;</p>	<p style="text-align: center;"><i>Measurements</i></p> <p>Length, height, width (empirical measurements);</p> <p>Time (name of the time – the concept of the duration of timeframes with respect to happened events)</p> <p>Money;</p> <p>Weight (mass);</p> <p>Patterns</p> <p style="text-align: center;">(22 hours)</p>	<p>Compare familiar magnitudes using known and arbitrary units of measurement; Put in order objects according to their: height, length, or width; Use expressions and events related to past, present, future, before, after, fast, slow, etc.;; Present the coins to the students and then students have to exchange coins according to coins' value; Buy things and add up their cost; Compare objects with the same volume but different mass i.e., paper plate and glass plate; Observe and be familiar making objects balance on a weight skill; Develop patterns of simple geometrical figures; Develop patterns to count up to 20 by twos or count from 20 down by twos.</p> <p><i>Measure different magnitudes in the classroom, in the school yard, etc., i.e. the game of architecture. Play other games of measurements (measure time – day timeframes, money-transactions)</i></p>
<p>Distinguish planar figures: triangles, squares, rectangles, circles, and solids: triangular pyramids, cubes, right rectangular prisms, cylinders, spheres; Construct line segments using straightedge and joining two points; Make simple puzzles; Place, locate, and move objects with respect to student's position or the position of immovable objects; Recognize and observe reflective symmetry in pictures and figures;</p>	<p style="text-align: center;"><i>Geometry</i></p> <p>Planar figures and solids; Recognize them; Draw them; Spatial orientation; Reflective symmetry;</p> <p style="text-align: center;">(8 hours)</p>	<p>Classify geometrical objects according to their shape, the number of their sides or angles; Connect 1, 2, ..., or 10 points using straightedge and forming sketches. Describe a path on graph paper or a path in space; Tile a surface using smaller surfaces; Use the expressions: up-down, before-behind, left-right;</p> <p><i>Find out solids around (buildings, objects); Play games with puzzles, straws and play dough constructing planar figures and solids.</i></p>

In total 120 hours

Proposed interdisciplinary worksheets:

Subject: Transportation

Students make projects in groups collecting the words that describe different vehicles of transportation and they count how frequently these words are used daily, monthly, and yearly. They count, draw, collect pictures of, and compare different vehicles of transportation. **Fundamental interdisciplinary concepts:** Space-time-similarity-difference-change-evolution-civilization.

Subject: What is it in my school bag?

Students count the number of objects that are in their school bag (i.e., books, notebooks, pencils, marks, etc.). They discuss about the made material of each object. They pick up an object, they draw similar objects, and they collect pictures of this object. Using the number of the objects they counted and working in groups, students construct problem situations using addition and subtraction. Then they solve the problems and they present the solutions in the classroom using posters, pictures they collected, or their constructions. **Fundamental interdisciplinary concepts:** Space-similarity-difference-quantity.

Subject: My toys

Students count and express the number of the games a) they use to play, and b) their parents used to play (they have collected relevant information). They distinguish how many of these games are team games and how many are individual games. Each group presents the most famous of these games in the classroom and if it is possible students play the game. **Fundamental interdisciplinary concepts:** Space-time-similarity-difference-civilization.

2nd Grade

1. Content Guiding Principles, Goals, and Fundamental Concepts of Interdisciplinary Approaches

I. ELEMENTARY SCHOOL

Grade	Content Guiding Principles	General Goals (knowledge, skills, attitudes, and values)	Indicative Fundamental Interdisciplinary Approaches
1 st , 2 nd , 3 rd , 4 th , 5 th , 6 th	Problem Solving	Students must be able to: explore mathematical situations; pose problems and formulate questions using a set of given data; rephrase problems; recognize and describe similar problems and mathematical situations; investigate open-ended questions; apply their mathematical skills in everyday situations; and learn how to use calculators, computers, internet, and other recourses	Change Interaction System Communication Individual-group (element-set) Similarity-Difference
2 st	Numbers and Operations	Students must be able to: count orally, read, write, and order natural numbers up to 1,000; add, subtract, and multiply numbers not greater than 100; use the commutative and associative property in addition and	Change Interaction System Civilization Individual-group

		multiplication; understand the concept of sharing equally in the division.	(element-set) Similarity-Difference
2 st	Measurements	measure length and surface with standard and non-standard units of measurement; practice to measure time, money, and mass; recognize, describe, extend, and generalize numeric and geometric patterns	Change System Space-Time Individual-group (element-set) Similarity-Difference
2 st	Geometry	draw and reproduce geometrical figures and recognize their geometrical features; define points and draw line segments and straight lines; recognize by experience parallel lines and perpendicular lines; identify solids: cubes, right rectangular prisms, cylinders, and spheres; recognize the line(s) of reflection in geometrical figures and reflect an image over a line of reflection.	Change System Space-Time Individual-group (element-set) Similarity-Difference

GREEK CURRICULUM IN MATHEMATICS

ELEMENTARY SCHOOL

1. Particular goals

Students might be able to:

Construct elementary mathematical knowledge and abilities;

Communicate using mathematical language;

Understand elementary mathematical methods;

Develop critical thinking processes and proving abilities;

Develop problem solving ability;

Apply and use mathematics in everyday problems;

Understand the evolution of mathematics (the historical evolution of mathematical tools, symbols, and concepts)

Develop positive attitude towards mathematics

2. Goals, Topics, and Indicative activities

2nd Grade

Goals	Topics (estimation of time)	Indicative activities
<p>In the beginning of the school year review activities must be solved and self-evaluation must be done by students.</p> <p>The review activities assist students to emerge, use, and stabilize the already existing knowledge and to build on the new knowledge without gaps;</p> <p>Explore mathematical situations relevant to the appropriate concepts of second grade;</p>	<p><i>Problem solving</i></p> <p><i>(18 hours)</i></p>	<p>Recognize similar or analogous problem to a given problem;</p> <p>Check of the problem solving process;</p> <p>Use students' thoughts and strategies in the problem solving process to express father thoughts;</p> <p><i>Solve problems through dramatization, games or about real life transactions.</i></p>

<p>Check and verify results of operations.</p> <p>Distinguish the given from what to figure out in a problem. Select the appropriate given information to solve the problem;</p> <p>Students self-evaluate the construction of their knowledge and the amelioration of their abilities in order to give feedback to their learning process;</p>		
<p>Students must be able to:</p> <p>group or exchange using tens, hundreds, and thousands;</p> <p>write and name the natural numbers up to 1,000;</p> <p>switch from verbal to symbolic presentation of the numbers and vice-versa;</p> <p>break down three-digit numbers in sums of tens, hundreds, and thousands;</p> <p>distinguish the different place value of each digit of a number (tens, hundreds, and thousands);</p>	<p><i>Numbers and operations</i></p> <p>Numbers from 100 up to 1,000;</p> <p>Read and write numbers in decimal system;</p> <p>(16 hours)</p>	<p>Use the number line and a book to read numbers;</p> <p><i>Use appropriate material in collections which could be in groups and regroups (cubes, sticks, abacus). For example the following problem could be given:</i></p> <p><i>“the problem of shepherd”</i></p> <p><i>The shepherd in order to count his sheep he puts a stone in a bag. Students have to find out a solution when the bag will be full or too heavy. Then each stone could present ten sheep;</i></p>
<p>compare two natural numbers and use the correct symbol of inequality;</p> <p>order natural numbers and locate natural numbers on the number line;</p> <p>insert one or two numbers between two other numbers when this is feasible;</p>	<p><i>Numbers and operations</i></p> <p>Order and compare natural numbers.</p> <p>(6 hours)</p>	<p>insert one or two numbers between two other numbers;</p> <p>Locate natural numbers on the number line.</p>
<p>recognize the coins and their relationships up to their level of numerical knowledge and get acquainted with situations of transactions;</p> <p>enrich their experiences solving real life problems by transactions.</p>	<p><i>Numbers and operations</i></p> <p>Money.</p> <p>(5 hours)</p>	<p><i>Construct and play games using euros i.e., they create a ladder using the subdivisions of euro and play using a die to practice the subdivisions of euro;</i></p> <p><i>Students collect or find pictures of coins of euro from different countries. They discuss about the symbols on money.</i></p>
<p>measure lengths and surfaces;</p> <p>measure and compare results of measurements in meter or its subdivisions;</p> <p>use the units of mass (kilo or kilogram, gram);</p>	<p><i>Measurements</i></p> <p>Measurements (length, surface, mass, time)</p>	<p>Measurement of surface using as units of measurement other smaller surfaces and geometrical figures (i.e., triangles, squares);</p> <p>to get acquainted with the function of a weight skill;</p>

<p>To get acquainted with the concept of time and be able to compare duration of timeframes (days of a week, months of a year, calendar);</p>	<p>(12 hours)</p>	<p><i>Students measure temperature, height, weight, time, the dimensions of notebooks, books, desks;</i></p> <p><i>Dramatize real life events (i.e., visit a grocery, daily or weekly business at home).</i></p>
<p>Recognize, describe, and expand arithmetic and geometric patterns.</p>	<p><i>Measurements</i> <i>Patterns</i> (4 hours)</p>	<p>Construct patterns using beads or other material;</p> <p>Form arithmetic patterns ascending or descending by twos, by threes, by fives, and by tens up to 100;</p> <p>Draw geometric patterns characteristics of different cultures.</p>
<p>Add and subtract one-digit numbers using doubles, fives and tens, mentally or writing;</p> <p>Memorize the usual techniques of the operations of addition and subtraction with two-digit and three-digit numbers with or without carried over;</p> <p>Switch horizontal additions and subtractions to vertical and do the operations (especially with numbers of different number of digits)</p>	<p><i>Numbers and operations</i></p> <p>Calculations: Addition and subtraction of natural numbers form 0 up to 100.</p> <p>(20 hours)</p>	<p>Use objects to count, figures to check the steps, meter to measure;</p> <p>Verbal process of counting (count up or count down);</p> <p>Count using appropriate groups of objects (groups of fives and of tens) (abacus of two colors);</p> <p>Use of doubles (i.e., calculate $9+8$ as $1+8+8$), calculate through ten (i.e., $9+8=(9+1)+7$), calculate through five [i.e., $9+8=(5+4)+(5+3)$];</p> <p>Use the abacus or any other teaching aid students analyze the numbers in tens and ones. For example $30+20=50$, $23+15=10+10+3+5$, $18-3=10+8-3$. Students add or subtract numbers that are multiplies of 10 (i.e., $78+10, 207-10, 600-400$). They add numbers like: $34+40$, $15+8$, $18+7$. They subtract i.e., $25-19$ using the method of complement (how many do you need from 19 up to 25?)</p> <p>They solve problems of increase using addition, and of decrease using subtraction.</p> <p><i>They play role games i.e., they assemble a bench in the classroom and play vendors and customers, etc.</i></p>
<p>Understand the operation of the multiplication as multiply addition. Learn the symbol of the</p>	<p><i>Numbers and operations</i></p> <p>Calculations (horizontal writing of multiplication, multiplication table,</p>	<p>Present examples of commutative property in multiplication (i.e., put students in 3×4 ranks and then in</p>

<p>multiplication. To get acquainted with the verbal practice of multiplication table and the written horizontal multiplications. Learn the commutative and associative property in addition and multiplication.</p>	<p>not standard algorithm of multiplication) The symbol “.”. <i>(15 hours)</i></p>	<p>4x3 ranks to observe that the number of students is the same) and then learn the multiplication table; <i>Students experience situations of repetition or equal quantities i.e., each student walks on a path back and forth to understand the concept of “times” carrying the same number of book.</i></p>
<p>To get acquainted with the concept of sharing equally in division.</p>	<p><i>Numbers and operations</i> Situations of sharing equally (not the algorithm of division) <i>(12 hours)</i></p>	<p><i>Students experience group situations of sharing (equal or unequal parts) and part equalization i.e., teeter-totter, “bee passes”(Greek game)</i></p>
<p>Recognize and define points, draw line segments and straight lines on white paper or on graph paper; Measure and compare the length of line segments using standard units of measurement; Recognize by experience parallel and perpendicular lines; recognize planar figures: circles, squares, rectangles, triangles; examine the characteristics of geometrical figures using geometrical tools; Distinguish solids: pyramids, cubes, right rectangular prisms, cylinders, spheres; Draw geometrical figures using straightedge on white paper or on graph paper and reproduce geometrical figures; Observe lines of reflection in geometrical figures; Construct line segments using straightedge and joining two points; Make simple puzzles; Place, locate, and move objects with respect to student’s position or the position of immovable objects; Recognize and observe reflective symmetry in pictures and figures;</p>	<p><i>Geometry</i> <i>(12 hours)</i></p>	<p>Identify nodes and squares on the Cartesian plane (graph paper, crossword puzzles, maps); Describe a path in space or on graph paper; Use of gnomon to check if two lines are perpendicular; Form simple figures using pieces of puzzle (tag ram). Tile a surface using smaller surfaces. Play tag ram, tessellate, puzzle, numeric or logic games, patterns; Verify using a ruler congruence of sides and using gnomon verify if the angles are right; Fold paper to check and complete symmetry; <i>Students play mathematical games i.e., ticktacktoe.</i></p>

In total 120 hours

Proposed interdisciplinary worksheets:

Subject: Four seasons Students make projects in groups collecting information, writing down, counting, drawing, and dramatizing activities at home relevant to each season and they create mathematical situations about shopping in

terms of each season. **Fundamental interdisciplinary concepts:** Space-time-similarity-difference-change-evolution-civilization.

Subject: 24 hours

Students in groups write down their daily schedule i.e., what time they wake up, how much time they need to be ready, how long they stay at school (school schedule), how many hours they study, play, or sleep. They fill in individual or in group cards including their answers; they discuss and create mathematical situations of addition and subtraction. They collect pictures relevant to their schedule and they draw activities they are doing in their schedule. **Fundamental interdisciplinary concepts:** Space-similarity-difference-quantity.

3rd Grade

1. Content Guiding Principles, Goals, and Fundamental Concepts of Interdisciplinary Approaches

I. ELEMENTARY SCHOOL

Grade	Content Guiding Principles	General Goals (knowledge, skills, attitudes, and values)	Indicative Fundamental Interdisciplinary Approaches
1 st , 2 nd , 3 rd , 4 th , 5 th , 6 th	Problem Solving	Students must be able to: explore mathematical situations; pose problems and formulate questions using a set of given data; rephrase problems; recognize and describe similar problems and mathematical situations; investigate open-ended questions; apply their mathematical skills in everyday situations; and learn how to use calculators, computers, internet, and other recourses	Change Interaction System Communication Individual-group (element-set) Similarity-Difference
3 rd	Numbers and Operations	Students must be able to: count orally, read, write, and order natural numbers up to 10,000; add and subtract natural numbers not greater than 1,000; develop number sense for fractions and decimals in the decimal system of numbers; become familiar with the algorithm of multiplication and of division of natural numbers.	Change Interaction System Civilization Individual-group (element-set) Similarity-Difference
3 rd	Measurements	Identify and use the measurement units of length, time, and mass; Recognize a pattern and find out that iteration process is infinite.	Change Interaction System Space-Time
3 rd	Geometry	get acquainted with description, reproduction, and drawing process of geometrical figures and solids as well as to draw perpendicular lines using geometrical tools; become familiar with the concepts of vertices, edges, right angles, and faces; practice on the construction of symmetric figures over a line of reflection.	System Space-Time Symmetry Individual-group (element-set) Similarity-Difference

GREEK CURRICULUM IN MATHEMATICS

ELEMENTARY SCHOOL

1. Particular goals

Students might be able to:
 Construct elementary mathematical knowledge and abilities;
 Communicate using mathematical language;
 Understand elementary mathematical methods;
 Develop critical thinking processes and proving abilities;
 Develop problem solving ability;
 Apply and use mathematics in everyday problems;
 Understand the evolution of mathematics (the historical evolution of mathematical tools, symbols, and concepts)
 Develop positive attitude towards mathematics

2. Goals, Topics, and Indicative activities

<u>3rd Grade</u>		
Goals	Topics (estimation of time)	Indicative activities
<p>In the beginning of the school year review activities must be solved and self-evaluation must be done by students.</p> <p>The review activities assist students to emerge, use, and stabilize the already existing knowledge and to build on the new knowledge without gaps;</p> <p>Work individually or in groups without guidance solving simple problems;</p> <p>Explore open-ended mathematical situations relevant to the appropriate concepts of third grade;</p> <p>Check and verify results of operations.</p> <p>Distinguish the given from what to figure out in a problem. Select the appropriate given information to solve the problem;</p> <p>Argue for the truth of a solution;</p> <p>Present in the classroom their solution explicitly describing their strategy of the solution and giving the result of the solution;</p> <p>Estimate solutions and figure out if there are more than one solution in a problem;</p> <p>Students self-evaluate the construction of their knowledge and the amelioration of their abilities in order to give feedback to their learning process;</p>	<p><i>Problem solving</i></p> <p>Problem solving</p> <p style="text-align: center;"><i>(18 hours)</i></p>	<p>Recognize similar or analogous problem to a given problem;</p> <p>Check of the problem solving process;</p> <p>Use students' thoughts and strategies in the problem solving process to express further thoughts;</p> <p>Innovative activities in problem solving process as follows:</p> <p>Make a chart, draw a diagram or a graph;</p> <p>Try to solve a problem using simpler numbers.</p>

<p>Students must be able to: group or exchange using tens, hundreds, and thousands; write and name the natural numbers up to 1,000; switch from verbal to symbolic presentation of the numbers and vice-versa; break down three-digit numbers in sums of tens, hundreds, and thousands; distinguish the different place value of each digit of a number (tens, hundreds, and thousands);</p>	<p><i>Numbers and operations</i> Decimal system; Recognize the natural numbers up to 1,000, up to 2,000, up to 3,000, ... and finally up to 10,000;</p> <p style="text-align: center;">(12 hours)</p>	<p>Count natural numbers up to 10,000 using different ways; Recognize the place value of the digits in a natural number;</p> <p><i>Budget of family expenses.</i></p>
<p>compare two natural numbers and if they are not equal put them in order from the smaller to the bigger and conversely; use numbers to locate a point on the number line.</p>	<p><i>Numbers and operations</i> Order and compare natural numbers. The symbols “>”, “<”, “=”.</p> <p style="text-align: center;">(4 hours)</p>	<p>insert one or two natural numbers between two other numbers; Locate natural numbers on the number line.</p>
<p>recognize the coins and their relationships up to their level of numerical knowledge and get acquainted with situations of transactions; enrich their experiences solving real life problems by transactions using money; write correctly decimal numbers; distinguish the place value of each digit in decimal numbers; add and subtract decimal numbers up to two decimal digits.</p>	<p><i>Numbers and operations</i> Money. (3 hours)</p> <p>Introduce decimal numbers (6 hours)</p>	<p>Estimate decimal numbers using prices of products familiar to the students; Use the number line for visual representation.</p> <p><i>Students visit the cafeteria in their school or a grocery and estimate the prices of the products in terms of their weekly pocket money.</i></p>
<p>Use the standard tools of measurements; Know the standard measurement units of length and surface; Know the standard measurement units of mass; Know the standard measurement units of time; Order magnitudes.</p>	<p style="text-align: center;"><i>Measurements</i></p> <p>Measurements (length, surface, mass, time)</p> <p style="text-align: center;">(6 hours)</p>	<p>Initially students use non-standard measurement units of length and surface. They calculate for example the perimeter of a polygon by sticks. Then they get acquainted with standard measurements units and they use standard measurement tools (meter, balance, and clock).</p> <p><i>Students measure the dimensions of: their classroom, their school, windows, desks, doors, etc. They time events and they weigh objects.</i></p>
<p>Add and subtract mentally or in written version; Add and subtract three-digit or</p>	<p style="text-align: center;"><i>Numbers and operations</i> Calculations with numbers from 0 up to 1,000</p>	<p>Add and subtract using bigger numbers. In subtraction emphasize the algorithm.</p>

<p>four-digit numbers with or without carried over;</p> <p>Switch horizontal additions to vertical and do the operations (especially using numbers with different number of digits);</p> <p>Switch horizontal subtractions to vertical and do the operations (especially when the minuend and the subtrahend have different number of digits) ;</p> <p>Recognize that addition and subtraction are inverse operations;</p> <p>Get acquainted with the properties of addition and subtraction.</p>	<p>(18 hours)</p>	<p>Calculate mentally. They could be assisted by appropriate teaching material.</p> <p>Develop their calculating processes in subtraction.</p> <p>Become familiar with subtractions that are impossible in the set of positive numbers;</p> <p>Verify addition using subtraction.</p> <p>Games with problem situations (i.e., who will arrive first up to one hundred according to the rule: each student adds a number from 1 to 9)</p>
<p>Recognize patterns of numbers like the Pascal's triangle and find out that this process is infinite;</p> <p>Double natural numbers and guess the following terms in the sequence;</p>	<p><i>Measurements</i> Patterns (4 hours)</p>	<p>Continue the pattern in Pascal's triangle</p> $ \begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \dots \end{array} $ <p>where only the first three lines are given;</p> <p>The rule of multiplication by 2 is given and the sequence of numbers 1, 2, 4, 8, 16, 32, and students have first to continue the sequence according to the given rule and second to calculate the 13th term of the sequence;</p>
<p>Students must stabilize and complete the verbal practice of mental multiplication (multiplication table) as well as the written horizontal multiplications;</p> <p>To get acquainted with the commutative and associative property in addition and multiplication.</p> <p>Become familiar with the algorithm of multiplication between natural numbers, two-digit number times one-digit number and two-digit number times two-digit number.</p> <p>Be able to use the rule of zeros, when they multiply by 10, 100, 1,000.</p>	<p><i>Numbers and operations</i> Calculations (initial introduction into the multiplication algorithm) (15 hours)</p>	<p><i>Students could write down how many hours they watch children's shows on TV per week, during the last ten days, or during the last ten weeks, etc.</i></p>
<p>Divide by one-digit divisor;</p>	<p><i>Numbers and operations</i> Introduction to division</p>	<p>Introduction in the written horizontal divisions (inverse of multiplication table) as well as oral divisions;</p> <p>Empirical methods of problem solving using division and</p>

	(15 hours)	development of the already existing knowledge of students; Algorithm of division using the other operations (addition, consecutive subtractions); To get acquainted with activities in division; <i>Organize a field trip and calculate the cost of the field trip in total and per student.</i>
Become familiar with the simple fractions (i.e., $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{5}$, $\frac{1}{10}$, etc.) Compare simple fractions using appropriate representations.	<i>Numbers and operations</i> Introduction to “simple” fractions (10 hours)	Introduction in the concept of fractions using appropriate representations or natural models like: the clock with the subdivisions, geometric figures with line(s) of reflection, the subdivisions of lengths, to cut an apple or a chocolate in equal pieces and present it on a paper.
Draw geometrical figures using geometrical tools; Reproduce, describe, and draw basic geometrical figures (rectangles, squares); Describe and reproduce basic geometrical solids (cubes, spheres); Reproduce the net of a cube, a right rectangular prism, tetragonal pyramid; Construct perpendicular lines using geometrical tools. To understand the concept of right angle; Reflect a geometrical figure over a line of reflection.	<i>Geometry</i> (9 hours)	Project on different solids (reproduction, description, and representation) using the concepts of face, vertex, edge; Projects on different planar geometrical figures using the concepts of side, straight line, line segment, midpoint, line, perpendicular, angle, right angle; Find out geometrical figures in a complex picture and count them; Play tag ram, tessellate, puzzle, sticklers, and magic squares; For the reflection student use the folding process to construct the reflective figure of a planar figure. Also students use plastic mirror to recognize line(s) of reflection in a planar figure. <i>Draw the classroom.</i>

In total 120 hours

Proposed interdisciplinary worksheets:

Subject: My house

Students locate geometric figures in their house. First, they make individual projects and then projects in groups comparing their houses with other types of houses (i.e., igloo). **Fundamental interdisciplinary concepts:** Space-time-similarity-difference-civilization.

Subject: Our library

Students discuss about the construction material of the library and its role writing a text. They count the books or other objects in the library and they write down the numbers. Categorize the books and develop problematic situations of addition, subtraction, or multiplication grouping the books like handbooks, fables, plants and animals etc. Students draw the library on a piece of paper according to their written text. **Fundamental interdisciplinary concepts:** Space- quantity-change-civilization.

INTERDISCIPLINARY GREEK CURRICULUM FRAMEWORK FOR MATHEMATICS

1. Teaching aim

The aim of teaching mathematics, which can be placed among the general aims of school education, is to facilitate the students' personal development and provide them with the necessary skills for their smooth social integration. Mathematics can help students develop structured and critical thinking abilities and improve their reasoning abilities of analysis, abstraction and generalization that will enable them to express their thoughts in a neat, clear, simple, and accurate way.

Mathematics also sharpens students' abilities of observation, self-concentration and persistence, stimulates their initiative, creative imagination and freethinking, and fosters their sense of harmony, order, and beauty.

Mathematics is a necessary tool in everyday life, especially at the workplace. Mathematics also has a significant contribution to the development of other scientific fields, especially in Technology, Economics, and Social Studies.

2. Content Guiding Principles, Goals, and Fundamental Concepts of Interdisciplinary Approaches

I. ELEMENTARY SCHOOL

Grade	Content Guiding Principles	General Goals (knowledge, skills, attitudes, and values)	Indicative Fundamental Interdisciplinary Approaches
1 st , 2 nd , 3 rd , 4 th , 5 th , 6 th	Problem Solving	Students must be able to: explore mathematical situations; pose problems and formulate questions using a set of given data; rephrase problems; recognize and describe similar problems and mathematical situations; investigate open-ended questions; apply their mathematical skills in everyday situations; and learn how to use calculators, computers, internet, and other recourses	Change Interaction System Communication Individual-group (element-set) Similarity-Difference
4 th	Numbers and Operations (1.01, 1.02, 1.03, 1.04, 1.05)	Students must be able to: count orally, read, write, and order natural numbers up to 1,000,000; add, subtract, and multiply natural numbers not greater than 1,000; perform operations with decimal numbers and decimal fractions	Change System Individual-group Similarity-Difference
4 th	Measurements (2.01, 2.02, 5.01(3), 5.02(3))	Measure length, area, time, money, mass, and estimate capacity; Convert units of measurement and practice addition and subtraction using compound numbers; become familiar with simple numerical and geometric patterns.	Change System Space-Time Culture Individual-group (element/set) Similarity-Difference
4 th	Geometry (2.01, 2.02, 3.02, 3.03)	Draw parallel and vertical lines as well as geometric shapes with the use of appropriate instruments; Calculate the perimeter of simple figures; Understand intuitively the concept of area; Construct reflective figures on grid paper.	System Space-Time Individual-group (element/set) Symmetry Similarity-Difference
4 th	Gathering and processing data (4.01, 4.04)	Practice collecting, classifying, representing, and interpreting data; develop an appreciation for the use of probability in the real world.	System Organization

GREEK CURRICULUM IN MATHEMATICS

ELEMENTARY SCHOOL

1. Particular goals

Students might be able to:

Construct elementary mathematical knowledge and abilities;

Communicate using mathematical language;

Understand elementary mathematical methods;

Develop critical thinking processes and proving abilities;

Develop problem solving ability;

Apply and use mathematics in everyday problems;

Understand the evolution of mathematics (the historical evolution of mathematical tools, symbols, and concepts)

Develop positive attitude towards mathematics

2. Goals, Topics, and Indicative activities

4th Grade

Goals	Topics (estimation of time)	Indicative activities
<p>In the beginning of the school year review activities must be solved and self-evaluation must be done by students.</p> <p>The review activities assist students to emerge, use, and stabilize the already existing knowledge and to build on the new knowledge without gaps;</p> <p>Work individually or in groups without guidance solving simple problems;</p> <p>Explore open-ended mathematical situations relevant to the appropriate concepts of fourth grade;</p> <p>Check and verify results of operations.</p> <p>Distinguish the given from what to figure out in a problem. Select the appropriate given information to solve the problem;</p> <p>Argue for the truth of a solution;</p> <p>Present in the classroom their solution explicitly describing their strategy of the solution and giving the result of the solution;</p> <p>Figure out appropriate questions helpful to the solution process;</p> <p>Put more questions by themselves in a given problem and construct similar problems;</p> <p>Students self-evaluate the construction of their knowledge and the amelioration of their</p>	<p><i>Problem solving</i></p> <p>Problem solving</p> <p><i>(18 hours)</i></p>	<p>Recognize similar or analogous problem to a given problem;</p> <p>Check of the problem solving process;</p> <p>Use students' thoughts and strategies in the problem solving process to express farther thoughts;</p> <p>Innovative activities in problem solving process as follows:</p> <p>Make a chart, draw a diagram or a graph;</p> <p>Try to solve a problem using simpler numbers.</p>

abilities in order to give feedback to their learning process;		
<p>Students must be able to:</p> <p>write and name the natural numbers up to 1,000,000; Connect the verbal with the symbolic presentation of numbers; [1.01a(3)]</p> <p>switch from verbal to symbolic presentation of the numbers and vice-versa; [1.01a(3)]</p> <p>distinguish the different place value of each digit of a number (tens, hundreds, thousands, etc.); [1.01b(3)]</p> <p>recognize the fundamental characteristics of decimal system (place value); [1.01b(3)]</p> <p>Apply the process of grouping or exchanging in tens, hundreds, thousands, etc. [1.01a(3)]</p> <p>Write the decimal expansion of a natural number [1.02b(3)]</p>	<p><i>Numbers and operations</i></p> <p>Decimal system;</p> <p>Recognize the natural numbers up to 10,000, up to 20,000, up to 30,000, ... then up to 100,000, up to 200,000, up to 300,000,.... and finally up to 1,000,000;</p> <p>(8 hours)</p>	<p>Count natural numbers up to 10,000, up to 100,000, and finally up to 1,000,000 using different ways;</p> <p>Calculate how many natural numbers are between two given natural numbers;</p> <p>Recognize the place value of the digits in a natural number e.g., in 77,777 or to write the next number of the numbers 40,000 and 714,999;</p> <p>Write the decimal expansion of numbers e.g., $237,654 = 2 \cdot 100,000 + 3 \cdot 10,000 + 7 \cdot 1,000 + 6 \cdot 100 + 5 \cdot 10 + 4$.</p>
<p>recognize the coins and their relationships and get acquainted with situations of transactions; [2.02(3)]</p> <p>enrich their experiences solving real life problems by transactions using money. [2.01(3)]</p>	<p><i>Measurements</i></p> <p>Money</p> <p>(2 hours)</p>	<p>Dramatize real life events by transactions using money.</p>
<p>Compare two natural numbers and if they are not equal order them from the less to the greater. [1.01b]</p>	<p><i>Numbers and operations</i></p> <p>Order of natural numbers;</p> <p>Compare natural numbers.</p> <p>(2 hours)</p>	<p>Inserts one or more natural numbers between two given natural numbers if it is possible.</p> <p>Put natural numbers on the number line.</p>
<p>To be able to use arbitrary measurement units; [2.02(3)]</p> <p>To be able to use standard tools of measurements; [2.02(3)]</p> <p>Know the standard measurement units of length, area, mass, volume, and time; [2.02(3)]</p> <p>Convert units of standard measurements; [2.02(3)]</p> <p>Estimate the concept of liter (l) and of (ml), their relationship and solve real problems. [2.02a(3)]</p> <p>Measure length or mass using the corresponding units of measurement and their subdivisions to understand the compound numbers; [2.02b(3)]</p>	<p><i>Measurements</i></p> <p>Measurements (length, surface, mass, time, capacity)</p> <p>(10 hours)</p>	<p>Estimation of lengths, areas, and capacities using arbitrary units of measurement; for example to estimate areas and capacities using squares and cubes.</p> <p><i>Comparison of capacity between a small container of natural juice and a big one (family size). Check their prices to figure out which one to buy.</i></p>

<p>To be able to measure lengths and masses and represent the results as a natural, compound, or decimal number; [2.02b(3), 2.02c(3)]</p> <p>Add and subtract compound numbers. (N/A)</p>		
<p>Recognize simple geometric patterns; [5.01(3)]</p> <p>Triple (quadruple etc) natural numbers and guess the following terms in the sequence; [5.02(3)]</p>	<p style="text-align: center;"><i>Measurements</i></p> <p>Patterns</p> <p style="text-align: center;">(4 hours)</p>	<p>Recognize the pattern of repetition of numbers in a figure (e.g., in Pascal’s triangle) and note that this process continues infinitely;</p> <p>Give to the students the rule “multiply by 3” and the sequence of the numbers 1, 3, 6, 9, 12, ... and it is asked to figure out the following number and find the 10th term of the sequence.</p>
<p>Apply the properties of addition, subtraction, and multiplication; [1.04(3)]</p> <p>Make simpler calculations using appropriate properties of the operations; [1.04(3)]</p> <p>To get acquainted with the table of multiplication by heart, to make “simple” multiplications “mentally”; [1.03(3)]</p> <p>Develop strategies mental calculations of products using the properties of the multiplication; [1.03(3)]</p> <p>Double, triple, and quadruple numbers using the table of the multiplication. [1.03(3)]</p> <p>Analyze natural numbers as products of many factors; [1.05]</p> <p>Analyze and re-synthesize a product of numbers, if it is possible, in order to calculate it easier using the properties of the multiplication; [1.05]</p> <p>Multiply a natural number by 10, 100, 1,000; (N/A)</p> <p>Recognize that addition and subtraction are inverse operations; [1.02c(3)]</p> <p>To be familiar with the properties of the operations; [1.02c(3)]</p> <p>Verify the results of addition, subtraction, and multiplication. [1.02a(3), 1.02c(3)]</p>	<p style="text-align: center;"><i>Numbers and operations</i></p> <p>Calculations (addition, subtraction, and multiplication of natural numbers)</p> <p style="text-align: center;">(25 hours)</p>	<p>Add and subtract using bigger numbers. In subtraction emphasize the algorithm.</p> <p>Calculate mentally. They could be assisted by appropriate teaching material.</p> <p>Develop their calculating processes in subtraction.</p> <p>Become familiar with subtractions that are impossible in the set of positive numbers;</p> <p>Verify addition using subtraction.</p> <p>Games with problem situations (i.e., who will arrive first up to one hundred according to the rule: each student adds a number from 1 to 9)</p>
<p>Students must: stabilize and complete the verbal practice of</p>	<p style="text-align: center;"><i>Numbers and operations</i></p>	<p>Practice to analyze a natural number in product of different factors</p>

<p>mental or written horizontal divisions (inverse of the multiplication); [1.02c(3), 1.02e(3)]</p> <p>To get acquainted with the multiplies of 2, of 5, and of 10; [1.02(3)]</p> <p>To get acquainted with the algorithm of Euclidean division of two one-digit or two-digit natural numbers; (N/A)</p> <p>To get acquainted with the formula $D=d*q+r$ $0\leq r<d$ and verify the result of the division by this formula; (N/A)</p> <p>Distinguish situations of division of measurement and division of partition; [1.07]</p> <p>Recognize that multiplication and division are inverse operations; [1.02e(3)]</p> <p>Verify the results of the division; [1.02c(3)]</p> <p>Use the method of reduction in unit. (N/A)</p>	<p>Calculations (introduction into the algorithm of Euclidean division)</p> <p>(15 hours)</p>	<p>verbally;</p> <p>Calculate the quotient of the division by 2, by 3, and by 4;</p> <p>The roots of the reduction in unit are in the verbal and cultural tradition. So have to have verbal practice;</p> <p><i>Organize and cost (in total and per person) e.g., sport or cultural event in the school.</i></p>
<p>Students must be able to:</p> <p>Use the usual rules of writing decimal numbers; [1.01a(3)]</p> <p>Distinguish the place value of each digit in a decimal number; [1.02b(3)]</p> <p>Switch from decimal number to decimal fraction and vice versa; [1.05e(3)]</p> <p>Expand a decimal number; [1.01a(3)]</p> <p>Round decimal fractions and decimal numbers in a calculator. (N/A)</p>	<p><i>Numbers and operations</i></p> <p><i>Decimal numbers and decimal fractions</i></p> <p>(14 hours)</p>	<p>Comprehension of the necessity of fractions and decimal numbers through real life problems dividing lengths;</p> <p>Use of calculators;</p> <p>Activities like $1.3 = 1.30 = 1.300$. Also between 1.3 and 1.4 or 1.30 and 1.40 there are other decimal numbers, e.g., 1.31, 1.32, , 1.39,</p>
<p>Compare two decimal numbers; [1.03c(3)]</p> <p>Order decimal numbers from the less to the greater and vice versa; [1.03c(3)]</p> <p>Insert decimal numbers or natural numbers between two given decimal numbers; [1.03c(3)]</p> <p>Put decimal numbers on the number line. [1.01a(3)]</p>	<p><i>Numbers and operations</i></p> <p>Order of decimal numbers</p> <p>Comparison of decimal numbers</p> <p>(2 hours)</p>	<p>Practice to distinguish that two decimal numbers could have the same integer part but different number of decimal digits.</p>
<p>Students must:</p> <p>Understand the technique of addition and subtraction of decimal</p>	<p><i>Numbers and operations</i></p> <p>Addition and subtraction of decimal</p>	<p><i>Dramatize the shopping process in the classroom using the Euro and its subdivisions.</i></p>

<p>numbers; [1.02a] Switch the horizontal representation of addition and subtraction into the vertical and carry out the operations; [102a]</p>	<p>numbers (5 hours)</p>	
<p>Draw geometrical figures and solids using geometrical tools; [3.01(3)] Estimate the concept of area; [2.01] Calculate the perimeter of planar figures; [2.01] Reproduce and describe basic geometrical solids; [3.01(3)] Reproduce the net of some solids; (n/a) Describe and draw the basic geometric planar figures; [3.01(3)] Draw parallel, perpendicular, and intersected lines; [3.02] Draw the distance of a point from a line and the distance between two parallel lines; [3.02] Draw the reflective figure of a given figure; [3.03a] Dilate a figure on grid paper according to a given line segment. [3.03b]</p>	<p><i>Geometry</i> (10 hours)</p>	<p>To be able to use: Protractor, ruler, right triangle, as well as tracing paper and grid paper; Use the accurate geometric vocabulary; Examination of triangle, of square, of rectangle, of rhombus, and of polygon; Recognize and work on the solids: cube, rectangle solid, and sphere; Reproduce the net of more complex solids than in third grade; When the net of a solid is given students must be able to construct the corresponding solid; Reflection of a figure; Recognize the axes of symmetry in a planar figure and be able to construct the axes of symmetry; Construct parallel and perpendicular lines using the appropriate geometric instruments; From a point construct the perpendicular, the parallel, or the intersected line to another line; Recognize figures inside a figure; Count the number of figures in a complex figure; <i>Games with the net of solids;</i> <i>Symmetry in the nature (e.g., leaves of trees) and in arts (e.g., paintings, buildings).</i></p>
<p>Collect, classifying, interpreting, and presenting data; [4.01] Interpret graphics. [4.01]</p>	<p><i>Gathering and processing data</i> Gathering of data (introduction in statistics) (5 hours)</p>	<p>Use and interpretation of histograms and iconographies; Students could work in groups to complete activities collecting, classifying, and interpreting data e.g., the kind of breakfast they prefer etc. <i>The climate of a place (rainfall, temperature); the effect on the fauna and the flora; collection, classification, presentation of data; interpretation of graphics.</i></p>

In total 120 hours

Proposed interdisciplinary worksheets:

Subject: A meal or a dissert

Students look for recipes for meals or disserts. They write down the quantity of ingredients using fractions when this is possible ($1/2$, $1/3$, $1/4$, $3/4$, etc). Students work in groups presenting the results by drawing, synthesizing pictures, graphics, and constructions. **Fundamental interdisciplinary concepts:** Space-time-similarity-difference-change-civilization.

Subject: The car

Students collect data in groups about the history of a car, the types, and the quote of each type. They write down using fundamental rules of driving safely and draw the cars they like. Create problems' situations using speed, time, distance, and they solve them. **Fundamental interdisciplinary concepts:** Space-time-change-civilization-evolution.

INTERDISCIPLINARY GREEK CURRICULUM FRAMEWORK FOR MATHEMATICS

1. Teaching aim

The aim of teaching mathematics, which can be placed among the general aims of school education, is to facilitate the students' personal development and provide them with the necessary skills for their smooth social integration. Mathematics can help students develop structured and critical thinking abilities and improve their reasoning abilities of analysis, abstraction and generalization that will enable them to express their thoughts in a neat, clear, simple, and accurate way.

Mathematics also sharpens students' abilities of observation, self-concentration and persistence, stimulates their initiative, creative imagination and freethinking, and fosters their sense of harmony, order, and beauty.

Mathematics is a necessary tool in everyday life, especially at the workplace. Mathematics also has a significant contribution to the development of other scientific fields, especially in Technology, Economics, and Social Studies.

2. Content Guiding Principles, Goals, and Fundamental Concepts of Interdisciplinary Approaches

I. ELEMENTARY SCHOOL

Grade	Content Guiding Principles	General Goals (knowledge, skills, attitudes, and values)	Indicative Fundamental Interdisciplinary Approaches
1 st , 2 nd , 3 rd , 4 th , 5 th , 6 th	Problem Solving	Students must be able to: explore mathematical situations; pose problems and formulate questions using a set of given data; rephrase problems; recognize and describe similar problems and mathematical situations; investigate open-ended questions; apply their mathematical skills in everyday situations; and learn how to use calculators, computers, internet, and other recourses	Change Interaction System Communication Individual-group (element-set) Similarity-Difference
5 th	Numbers and Operations	Students must be able to: count orally, read, write, and order natural numbers up to 1,000,000,000 as well as fractions and rational numbers; add, subtract, multiply, and divide natural numbers, fractions, and rational numbers; add and subtract compound numbers; calculate the multiplies of 2, 3, 4, 5, ..., 10 and know which	Change System Individual-group Similarity-Difference

		numbers could be divided by 2, 5, and 10.	
5 th	Measurements	To stabilize their knowledge about the formal measurement units for length, mass, time, area, and capacity and be familiar with the use of those measurements in everyday life; To recognize, to describe, and to expand simple numerical and geometric patterns.	Change System Space-Time Culture Similarity-Difference
5 th	Geometry	Draw geometric shapes using appropriate instruments; Calculate the perimeter and the area of basic geometric shapes as well as the circumference; Name, classify, and construct different types of angles and triangles; Practice constructing the net of simple geometric solids.	Change System Space-Time Individual-group (element/set) Similarity-Difference Classification
5 th	Gathering and processing data	To introduce the concept of ordered pairs; Practice reading and constructing histograms, pictographs, and graph, as well as organizing data in tables; To become familiar with the concept of probability, forecasting, and calculating the average.	Change System Space-Time Individual-group (element/set) Similarity-Difference Probability

GREEK CURRICULUM IN MATHEMATICS

ELEMENTARY SCHOOL

1. Particular goals

Students might be able to:

Construct elementary mathematical knowledge and abilities;

Communicate using mathematical language;

Understand elementary mathematical methods;

Develop critical thinking processes and proving abilities;

Develop problem solving ability;

Apply and use mathematics in everyday problems;

Understand the evolution of mathematics (the historical evolution of mathematical tools, symbols, and concepts)

Develop positive attitude towards mathematics

2. Goals, Topics, and Indicative activities

5th Grade

Goals	Topics (estimation of time)	Indicative activities
<p>In the beginning of the school year review activities must be solved in 1. Numbers and operations, 2. Measurements, 3. Geometry, 4. Gathering and processing data and self-evaluation must be done by students.</p> <p>The review activities assist students to emerge, use, and stabilize the already existing knowledge and to build on the new knowledge and concepts without gaps;</p> <p>Work individually or in groups without guidance solving simple problems;</p> <p>Explore open-ended mathematical</p>	<p><i>Problem solving</i></p> <p>Problem solving</p> <p>(26 hours)</p>	<p>Recognize similar or analogous problem to a given problem;</p> <p>Check of the problem solving process;</p> <p>Use students' thoughts and strategies in the problem solving process;</p> <p>Innovative activities in problem solving process as follows:</p> <p>Make a chart, draw a diagram or a graph;</p> <p>Try to solve a problem using simpler numbers and examining specific cases;</p> <p>Figure out a model;</p> <p>Estimate and check;</p> <p>Token cross thematic activities in</p>

<p>situations relevant to the appropriate concepts of fifth grade; Check and verify results of operations.</p> <p>Distinguish the given from what to figure out in a problem. Select the appropriate given information to solve the problem;</p> <p>Argue for the truth of a solution;</p> <p>Figure out appropriate questions helpful to the solution process;</p> <p>Present in the classroom their solution explicitly describing their strategy of the solution and giving the result of the solution;</p> <p>Students self-evaluate the construction of their knowledge and the amelioration of their abilities in order to give feedback to their learning process;</p> <p>To have the opportunity to use computer programs like Sketchpad, Cabri, Logo, Word, Excel, Paint etc. to facilitate the solution of problems.</p>		<p>the problem solving process will be presented in each content guiding principle.</p>
<p>Students must be able to: write and name the natural numbers up to 1,000,000,000; Connect the verbal with the symbolic presentation of numbers; switch from verbal to symbolic presentation of the numbers and vice-versa; recognize the fundamental characteristics of decimal system (place value); express a number as sum or product of two other numbers;</p>	<p><i>Numbers and operations</i></p> <p>Decimal system;</p> <p>Recognize the natural numbers initially up to 10,000,000 20,000,000 ..., 100,000,000, 200,000,000 ..., and finally up to 1,000,000,000;</p> <p>Verbal and written representation (12 hours)</p>	<p>Count natural numbers up to 1,000,000,000 using different ways; Calculate how many natural numbers are between two given natural numbers;</p> <p><i>Express in meters the dimensions of the earth and the distances between the capitals of different states.</i> (Geography, Language Arts)</p>
<p>Compare two natural numbers and use appropriately the symbols of comparison;</p> <p>Compare two natural numbers and if they are not equal order them from the less to the greater and vice-versa;</p> <p>Put natural numbers on a number-line;</p> <p>Insert one or more natural numbers between two other natural numbers if it is possible.</p>	<p><i>Numbers and operations</i></p> <p>Order natural numbers;</p> <p>Compare natural numbers. (4 hours)</p>	<p><i>Write down the population of big states and continents.</i></p> <p><i>Compare, draw, and order the numbers on the number-line.</i> (Geography, Esthetic education)</p>
<p>Know the properties of the operations;</p> <p>Be able to use the properties of the operations in order to solve complex problems;</p> <p>Examine the result of a division</p>	<p><i>Numbers and operations</i></p> <p>Methods of exact calculations (addition, subtraction, multiplication, and division of natural numbers). (8 hours)</p>	<p>Complete miss numbers in equations or correct wrong algorithms to develop deeper knowledge about the operations and their properties.</p>

using the Euclidean formula $D=dq+r, 0 \leq r < d$; Solve problems with solution(s) rational number(s).		
Learn to use the calculator.	<i>Numbers and operations</i> The use of calculator. (1 hour)	The calculator could be used according to the instructor's decision: to solve problems with difficult calculations, for quick calculations, and when students have to pay attention on the process of solving problems. In addition, to verify mental or written calculations.
Approximate natural numbers when it is possible; Estimate the result of an operation.	<i>Numbers and operations</i> Methods of estimation and approximation. (4 hours)	Approximate natural numbers to estimate if the result is correct or not. <i>Write down the area of big countries and continents. Approximate the numbers, compare them, and put them in order on the number-line.</i>
Write down correctly the decimal numbers; Make sense of the place value of each digit of a decimal number; Be able to compare two decimal numbers using appropriately the symbols of comparison; be aware that two decimal numbers could have the same whole number part but different amount of decimal digits in the decimal part; Order decimal numbers from the less to the greater and vice-versa; Insert decimal numbers between two other decimal numbers or between natural numbers; Use decimal numbers to figure out positions on the number-line.	<i>Numbers and operations</i> Decimal numbers: write, name, and order them. (5 hours)	Understand that the inequality $13,7 < 13,475$ is wrong because the place value matters and not the amount of decimal digits; Insert a decimal number between two other decimal numbers. <i>In the classroom, weight different objects with the same volume but different mass. Write down the weights, compare them, and put the numbers in order on the number-line (Physics).</i>
Be able to add and subtract decimal numbers correctly; Be able to multiply a natural number or a decimal number by 10, 100, 1000, and 0.1, 0.01, 0.001; Be able to multiply a decimal number by a natural number as well as two decimal numbers; Be able to divide natural numbers and decimal numbers by a natural number; Be able to solve complex problems using addition and subtraction; Be able to link the decimal numbers with the meter.	<i>Numbers and operations</i> Decimal numbers: operations. (14 hours)	Mental calculations and approximate estimations using appropriate representations. <i>Figure out how the prices of products are written in order to understand the decimal written expression of the fractions of euro; Games with numbers i.e., find out two numbers with product 33333 (31111) etc. (Language Arts).</i>
Calculate the multiples of 2, 3, 4, 5, ..., 10; Calculate the LCD; Learn which numbers are divided	<i>Numbers and operations</i> Divisibility, multiples (4 hours)	Figure out the LCM by the method of multiples and by the method of prime factors.

by 2, by 5, and by 10;		
Students have to be able to: Create and distinguish fractions with the same denominator and fractions with different denominators; Be able to make fractions with common denominators; Simplify fractions; Compare and order fractions; Transform fractions in mixed numbers.	<i>Numbers and operations</i> Fractions (6 hours)	Figure out the relations between fractions using appropriate manipulative. Represent mixed numbers using the symbol of “and”. Simplify fractions using common divisors.
Students have to be able to: Add and subtract fractions; Multiply and divide fractions; Solve simple problems using fractions; Be able to use the method of calculating the unit fraction.	<i>Numbers and operations</i> Operations with fractions (15 hours)	Use appropriate representations for the multiplication and division of fractions; The method of unit fraction facilitates the understanding of multiplication and division of fractions; Different representations of equivalent fractions; <i>The fractions in music (halves, fourths, eighths) (Music).</i>
Be able to use the usual instruments of measuring (straightedge, protractor, right angle, meter, tape measure, balance, clock, and timer). Measure angles using as unit the $\frac{1}{2}$ of right angle and the $\frac{1}{4}$ of right angle; Convert standard units of length, area, time, and mass; be able to order the measurements and use the appropriate unit in each case; be able to express the measurements as a natural number, a compound number, and as a decimal number; be able to do operations with compound numbers; Use their monetary experiences solving real life problems.	<i>Measurements</i> Measurements (length, mass, time, area, angle, capacity, money) (5 hours)	In the beginning to experiment using non-standard units; Practice doing operations with compound numbers; Use of fractions and decimal numbers solving problems measuring length, mass, area, and time; <i>Measurements from antiquity up to nowadays. Units used in the antiquity – empirical units of measurement. The history of the institution of meter as unit of measurement. (History, language Arts).</i>
Recognize patterns of numbers, i.e., Pascal’s triangle, and find out that the process continues infinitely ; Be able to quadruple natural numbers and predict the following terms in the sequence.	<i>Measurements</i> Patterns (3 hours)	Students color the natural numbers that are less than 50 and multiplies of 2, 3, 8, and 9 on the number-line or i.e., in the Pascal’s triangle; Give students the rule “multiply by 4” and the series of numbers 1, 4, 16, 64, ... and ask them to continue the sequence of numbers according to the previous rule and figure out the 8 th term.
Draw geometrical figures using geometric instruments; Recognize shapes as inside parts of other complex shapes;	<i>Geometry</i> (8 hours)	Learn the terminology of shapes like straight line, circle, center, radius, diameter, angle; Analyze a complex geometric figure

<p>Calculate area of a square, of a rectangle, and of a right triangle; Compare areas; Distinguish the concept of area from the concept of perimeter solving problems where when the one concept is given to figure out the other concept; Calculate the circumference; Recognize the different types of angles (right, acute, obtuse); compare and draw angles; Distinguish the different types of triangles and their properties; Draw the altitudes of triangles using geometric instruments; Scale up and down simple shapes with straight lines on grid paper; Construct the symmetric figure of a given shape about a line of symmetry on grid paper;</p>		<p>making assumptions about its parts and verifying the assumptions using geometric instruments; Scale up and down figures on grid paper measuring them and not using scales and proportions; As an activity to construct symmetric figures; <i>Use mathematic concepts drawing wheel of bicycle using geometric software (Sketchpad, Cabri)</i> <i>(Language Arts, New technologies)</i></p>
<p>Gather t and record data; Put data in tables (frequency, diagrams, graphs, pictorial representation, histograms); Graph verbal or written data and vice-versa; Calculate the average of a set of data.</p>	<p><i>Gathering and processing data</i> <i>Statistics</i> Statistics <i>(5 hours)</i></p>	<p>Students explore a statistical problem relevant to their experiences; They collect data and they present them using statistical presentations. <i>Collect data how many times per day they laugh in the classroom and at home, make the graph of the data, calculate the average of the data</i> <i>(Language Arts, Social education).</i></p>

In total 120 hours

Proposed interdisciplinary worksheets:

Subject: Vehicular Accidents. Students work in groups and they collect data about the number of vehicular accidents during the last i.e., five years. They put the data in tables (frequency in percentage or in absolute numbers, graph, and histogram), they calculate the average of the data per month and per year. They refer causes and sequences. They develop discussion about the need of configuration of right driving behavior.

Fundamental interdisciplinary concepts: Interaction, similarity, difference, change, civilization.

Make projections into Esthetic education, Language Arts, and Social education.

Subject: Symmetry in our life. Students find symmetric figures in different constructions around them or in the past (i.e., houses in ancient Greece, in Middle Ages etc., buildings, carpet, wallpapers, cloths, tiles, etc). They present them in groups and they discuss the role and the type of symmetry in different cases.

Fundamental interdisciplinary concepts: Space, time, change, civilization, evolution.

Make projections into Esthetic education, Language Arts, Social education and History.

Subject: A giant in the classroom. Students work in groups and each group constructs big triangles, quadrilaterals, pentagons etc., and combine them in order to construct giants in the classroom. Then each group cuts paper ribbons in the shape of triangle, square, rectangle, hexagon, etc. and join them using prongs. Then they measure the perimeter and the area of each construction. They find general formulas to calculate the perimeter and the area of their constructions. They explain where such shapes exist in real life objects.

Fundamental interdisciplinary concepts: Space, change, system, similarity difference.

Make projections into Computer science, Esthetic education, Language Arts, Social education, and Explore physical world.